Comparison of Two Means, paired data

In the previous session, we learned how to do a significance test for a single mean in which we compared an observed mean from a sample with a hypothesized mean. In this session we learn how to compare two observed means i.e. two separate means which we have. There are two ways to obtain two means 1) paired data, 2) unpaired data.

Paired data

In paired data, one mean is paired with another mean. This happens when we have done repeated observations on the same individuals. For example if we study the effect of a drug on lowering blood sugar in a sample of 100 people; we measure blood sugar in each person before giving the drug and another time after the drug. These readings are paired. We compare the mean of the first reading with the mean of the second reading. This is paired data. The two samples are dependent on each other they are the same individuals each with two readings.

Unpaired data

In unpaired data, the two means are not from the same individuals; the two samples are independent on each other. For example if we wanted to compare the mean marks of a sample of boys and a sample of girls in biostatistics examination. Another example is when we want to study the effect of a drug on lowering blood sugar, we compare the mean blood sugar of people receiving the drug with the blood sugar of another group of people not receiving the drug.

Analysis of Paired Data

When analyzing paired data the first step is to calculate the difference between the two individual observations in each pair.

Example

We measured effect of a drug on systolic blood pleasure in group of 14 persons. We took two readings from each individual, one before giving the drug and one after giving the drug. Results are tabulated here.

ID	SBP before	SBP after
1	150	145
2	145	140
3	155	140
4	160	155
5	150	140
6	170	170
7	165	165
8	175	170
9	155	150
10	155	150
11	160	145
12	160	140
13	170	165
14	150	150

To analyze this data in order to see whether there is a difference in SBP in patients before and after using the drug we

1) Calculate difference in the two readings of SBP in each individual (see next table). To calculate the differences we subtract SBP measured before the drug from SBP after the drug.
2) Then we calculate the mean of this difference (mean difference, δ) and its standard deviation (see bottom of next table).
3) We calculate 95% confidence interval for this mean difference.
4) Finally we perform the hypothesis test

The mean difference of SBP between the two readings is 6.8 mmHg. What does this mean? This mean that on average there is 6.8 mm reduction in SBP after using the drug. But is this difference statistically significant or it is due to chance (random variation)? To answer this question, we have to measure 95% CI for the mean difference and then do a hypothesis test.

Confidence Interval for Mean Difference

In order to calculate a 95% CI for the sample mean difference we need

- The mean difference
- The standard error of the mean difference

Note: we use the standard deviation and sample size to calculate the standard error

We calculate 95% CI of the mean difference using the following formula:

Mean difference ± 1.96 x standard error of the mean difference

ID	SBP before	SBP after	difference in SBP
1	150	145	5
2	145	140	5
3	155	140	15
4	160	155	5
5	150	140	10
6	170	170	0
7	165	165	0
8	175	170	5
9	155	150	5
10	155	150	5
11	160	145	15
12	160	140	20
13	170	165	5
14	150	150	0
Mean difference in SBP			**6.8**
SD of mean difference			**6.1**

$$\bar{d} \pm 1.96 \times s_{\bar{d}} / \sqrt{n}$$

Mean difference in SBP=6.8
SD=6.1
SE= SD/√n
=6.1/√14=1.63
.
Now we can calculate 95% CI for the mean difference

Lower limit:
Mean difference – 1.96 x SE=6.8- 1.96x1.63=6.8 – 3.2=3.6 mmHg

Upper limit:
Mean difference + 1.96 x SE=6.8 + 1.96 x1.63=6.8 + 3.2=10.0 mmHg
Therefore the 95% CI of the mean difference is from 3.6 mmHg to 10 mm Hg. What does this mean? This means that we can be sure 95% that the mean difference between the two readings of SBP which were taken before and after the drug falls somewhere between 3.6 to 10 mm Hg. In other words the drug leads to a reduction in SBP in 95% of the population by an average which ranges from 3.6 to 10 mmHg.

Hypothesis Test for a Mean Difference

We do a hypothesis test to check whether the difference between the mean SBP before and after the drug is statistically significant or it is due to random variation. First we have to formulate the null and the alternative hypothesis:

The **Null Hypothesis** is that the mean SBP of the two samples are similar i.e. the mean difference is zero:

$$\mathbf{H_0 : \delta = 0}$$

The **Alternative Hypothesis** is that the two means are truly different i.e. the mean difference of SBP is not zero:

$$\mathbf{H_1 : \delta \neq 0}$$

We can use either z test or t test. If the sample is big we use z test. But in our example of 14 patients, the sample is small therefore we use t test.

$$t = \frac{\text{sample mean difference} - \text{hypothesised mean difference}}{\text{standard error of the mean difference}}$$

The mean difference is 6.8
The hypothesized difference is zero
SE= 1.63

t=(6.8-0)/1.63=4.2

We look for the probability (P value) corresponding to t=4.2 in a table of t- distribution on 13 degrees of freedom (*df*). We can see from the table that 4.2 corresponds to a p value of 0.001. This probability is very small and could not be due to random error. Therefore we reject the null hypothesis that there is no difference and accept the alternative hypothesis. This means that there is a true, statistically significant difference, between the mean SBP before the drug and after the drug. In other words this indicates that the drug does reduce blood pleasure and this effect is true, it is not due to chance.

Table A2 *t*-distribution.

	Two-tailed *P*-value			
df	0.10	0.05	0.01	0.001
1	6.314	12.706	63.656	636.58
2	2.920	4.303	9.925	31.600
3	2.353	3.182	5.841	12.924
4	2.132	2.776	4.604	8.610
5	2.015	2.571	4.032	6.869
6	1.943	2.447	3.707	5.959
7	1.895	2.365	3.499	5.408
8	1.860	2.306	3.355	5.041
9	1.833	2.262	3.250	4.781
10	1.812	2.228	3.169	4.587
11	1.796	2.201	3.106	4.437
12	1.782	2.179	3.055	4.318
13	1.771	2.160	3.012	4.221
14	1.761	2.145	2.977	4.140
15	1.753	2.131	2.947	4.073
16	1.746	2.120	2.921	4.015
17	1.740	2.110	2.898	3.965