Comparison of Two Means

In the previous session, we learned how to do a significance test for a single mean in which we compared an observed mean from a sample with a hypothesized mean. In this session we learn how to compare two observed means i.e. two separate means which we have. These two means are obtained in unpaired data.

Unpaired data

In unpaired data, the two means are not from the same individuals; the two samples are independent on each other. For example if we wanted to compare the mean marks of boys and a girls in epidemiology examination at the nursing school. Another example is when we want to study the effect of a drug on lowering blood sugar, we compare the mean blood sugar of people receiving the drug with the blood sugar of another group of people not receiving the drug.

Student	Sex	Mark
1.	Female	90
2.	Female	67
3.	Male	75
4.	Male	78
5.	Female	54
6.	Male	91
7.	Male	51
8.	Female	51
9.	Female	51
10.	Female	66
11.	Female	79
12.	Female	56
13.	Female	72
14.	Female	81
15.	Male	78
16.	Female	81
17.	Male	86
18.	Female	85
19.	Male	83
20.	Female	67
21.	Female	65
22.	Female	90
23.	Male	76
24.	Female	56
25.	Female	75
26.	Female	81
27.	Female	58
28.	Female	85
29.	Female	94
Mean mark		**73.3**
SD		**13.4**

Analysis of unpaired data (independent means)

When analyzing unpaired data the first step is to calculate the difference between the two individual observations in each pair.

Example

The table opposite shows the epidemiology examination marks of students at the Nursing College, university of Sulaimani. We want to know whether there is any significant difference between the marks of boys and girls. In other words are boys doing significantly better than girls in the examination (or vice versa); or the boys and girls have got similar marks?

To do a hypothesis test for this data we have to

1) Calculate the mean and SD for boys and girls
2) Calculate difference in the two means and the standard error for the difference
3) Then we calculate the 95% confidence interval for this mean difference.
4) Finally we perform the hypothesis test

	Sample size (n_1,n_2)	**Mean marks (x_1, x_2)**	**SD (s_1, s_2)**	**SE**
Boys	8	77.3	11.9	4.2
Girls	21	71.6	13.8	3.0
Mean difference in marks		5.7		5.5

Note: for a large sample, the standard error of the mean difference is calculated using the formula below:

$$\mathrm{SE}(\bar{x}_1 - \bar{x}_2) = \sqrt{s_1^{\,2}/n_1 + s_2^{\,2}/n_2}$$

But if the sample is small, we calculate the common variance (an average of the two sample variances) from the formula and then calculate the SE of the difference using the other formula.

$$s^2 = \frac{(n_1-1)s_1^{\,2} + (n_2-1)s_2^{\,2}}{(n_1-1)+(n_2-1)} \qquad \mathrm{SE}(\bar{x}_1 - \bar{x}_2) = s \times \sqrt{(1/n_1 + 1/n_2)}$$

Confidence Interval for the Difference of Two Independent Means

The 95% CI for the mean difference is given by

Difference in means $\pm$ 1.96 SE (difference in means)

The full formula will be this

$$\bar{x}_1 - \bar{x}_2 \pm 1.96\sqrt{s_1^{\,2}/n_1 + s_2^{\,2}/n_2}$$

For comparison of the marks of boys and girls:

The mean difference = 5.7
The standard error = 5.5

The lower limit of the 95% CI is

5.7 – 1.96 X 5.5=5.7-10.8= -5.1

The upper limit of the 95% CI is

5.7 + 1.96 X 5.5=5.7 + 10.8= 16.5

Therefore the 95% CI for the mean difference in marks between boys and girls is from -5.1 to 16.5. This difference is very wide and includes zero (no difference) because it ranges from minus 5 to plus 16.5. when the difference between two means includes zero, it indicates that there is no significant difference between the two means. The difference is most probably due to chance.

Hypothesis Test for the Difference in Means

We do a hypothesis test to check whether the difference between the difference in mean marks of boys and girls are statistically significant or it is due to random variation. First we have to formulate the null and the alternative hypothesis:

The **Null Hypothesis** is that the mean marks the two samples are similar i.e. the mean difference is zero:

$H_0 : \delta = 0$ i.e. $x_1 - x_2 = 0$

The **Alternative Hypothesis** is that the two means are truly different i.e. the mean difference of SBP is not zero:

$\mathbf{H_1 : \delta \neq 0}$ **i.e.** $\mathbf{x_1 - x_2 \neq 0}$

We can use either z test or t test. If the sample is big we use z test. But in our example of 29 students, the sample is small therefore we use t test given by the following formula

t= (X1- X2)/ SE(mean difference)

Mean difference (**X1- X2)= 5.5**
SE of the mean difference =5.5

t= 5.7/5.5=1.04

In the table of t distribution we look for the p value corresponding to t=1.04 on 28 degrees of freedom. On 28 df we don't see 1.04. We look for the nearest figure to 1.04 which is 1.7. This t value corresponds to p=0.1 or 10%. Since our figure of 1.7 is bigger than 1.7 the p value will be larger than 0.1. The actual p given by statistical software is 0.3. This result means that the probability of finding the observed difference by chance is very large (30%), therefore we accept the null hypothesis which says that the mean marks of boys and girls are not different (are similar) because there is a 30% chance that we can get the observed difference of 5.7 by chance. In conclusion we say that there is no statistically significant difference between the mean marks of boys and girls in the epidemiology examination at the nursing school. The boys and girls have achieved similar results.

Table A2 *t*-distribution.

	Two-tailed *P*-value			
df	0.10	0.05	0.01	0.001
1	6.314	12.706	63.656	636.58
2	2.920	4.303	9.925	31.600
3	2.353	3.182	5.841	12.924
4	2.132	2.776	4.604	8.610
5	2.015	2.571	4.032	6.869
6	1.943	2.447	3.707	5.959
7	1.895	2.365	3.499	5.408
8	1.860	2.306	3.355	5.041
9	1.833	2.262	3.250	4.781
10	1.812	2.228	3.169	4.587
11	1.796	2.201	3.106	4.437
12	1.782	2.179	3.055	4.318
13	1.771	2.160	3.012	4.221
14	1.761	2.145	2.977	4.140
15	1.753	2.131	2.947	4.073
16	1.746	2.120	2.921	4.015
17	1.740	2.110	2.898	3.965
18	1.734	2.101	2.878	3.922
19	1.729	2.093	2.861	3.883
20	1.725	2.086	2.845	3.850
21	1.721	2.080	2.831	3.819
22	1.717	2.074	2.819	3.792
23	1.714	2.069	2.807	3.768
24	1.711	2.064	2.797	3.745
25	1.708	2.060	2.787	3.725
26	1.706	2.056	2.779	3.707
27	1.703	2.052	2.771	3.689
28	1.701	2.048	2.763	3.674
29	1.699	2.045	2.756	3.660

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