PROBABILITY

The theory of probability originated with players of cards and dice in the seventeenth and eighteenth centuries. Since a die has 6 faces (1,2,3,4,5,6), the probability of getting a 3 when we throw one die is one in six. We write this as:

prob(3)= 1/ 6 = 0.167 = 16.7%

This result tells us that if we throw a die 6 times, one time we get 3, because there are 6 faces in a die and only one face is numbered 3. But this is in theory. However, if we actually throw a die six times, we may not get a 3, or we may even get a 3 twice or even more. If we throw the die many more times, say a hundred times, we will get closer to the theoretical result and get around 16 times each of the 6 faces of the die. Therefore we could define probability in this way:

The probability that an <u>event</u> will happen is the <u>proportion</u> of times that we would observe it if we repeated the <u>experiment</u> a large number of times. The *probability is the proportion of times that the event would happen in the long run.*

The probability is a proportion between 0 and 1. The smallest probability of an event is zero and the largest probability is one. It is the number of times the event happens divided by total number of times the experiment is repeated. The reason why we need a large number of times to get closer to the theoretical probability is chance or what we call random variation. If we repeat an experiment a small number of times, the result will not be reliable due to random variation. The larger the number of experiments (e.g. sample size) the less is the role of random variation. However, even in very big sample size the role of chance is not totally eliminated. For example even if we throw a die 1000 times we may not exactly get 167 time each of the faces of the die because chance still could play a role.

Probability laws (rules)

If we are talking about one event, the probability is simply a proportion of the total. But when we talk about two or more events the probability becomes more complicated and cannot be calculated by a simple division on total. For these situations, we have two fundamental laws of probability: 1) the **additive law** and 2) the **multiplicative law**

The Additive Law

The events that cannot happen at the same time are called mutually exclusive events i.e. one excludes the other which means that one cannot happen if the other happens. In other words either event A happens or event B. Both events cannot happen together. In this situation we calculate the probability by summing the probability of both events (the additive law).

For example what is the probability of getting **<u>either</u>** 3 <u>or</u> 6 in a throw of a die. When we throw a die we cannot have both 3 and 6 at one throw, therefore these events are mutually exclusive and the probability if calculated by the additive law which says:.

Probability (A or B)= prob(A) + prob(B)

Therefore the probability of either 3 or 6 in a thorw (prob 3 or 6)= prob(3) + prob(6) =1/6+1/6=1/3=33%

Now consider this example: suppose a box contains 50 balls; 10 red, 10 blue, 30 green.

What is the probability of picking a red ball?

prob (red) = 10/50= 1/5=20%

What is the probability of picking a green ball?

prob (green) = 30/50= 60%

What is the probability of picking <u>a</u> red <u>or</u> a green ball?
Using the additive law:

prob (red or green) = prob(red) + prob(green) = 20%+ 60% =80%

What is the probability of picking a ball of any color?
1 or 100%

What is the probability of picking a ball that is neither red, nor green?
Probability of either red or green=80% and probability of all colors is 100% therefore the probability of neither red nor green =100%-80%= 20%

The Multiplicative Law
Suppose that we have **two** boxes, each containing 50 balls of different colors like the box above. In this case if we draw a ball from one box, and a ball from the other box, the color of the two balls will not be dependant on each other, i.e. they will not be mutually exclusive because we can draw a red ball from box one and a red ball from box B. Whatever color we pick from box A, the probability of picking any color from box B will not be affected. In other words we can draw any color from either box. The two events will not be mutually exclusive.

So if we draw one ball from each box, what is the probability of picking 2 red balls (a red ball from box A <u>**and**</u> a red ball from box B) ? This question is answered using the multiplicative law. The multiplicative law states that the probability of two <u>independent</u> events is given by the **product** of their individual probabilities. So:

prob(*A and B*) = prob(*A*) x prob(*B*)

Prob (red) box A=10/50=20%
Prob (red) box B=10/50=20%
Probability (A+B)= prob(A) X prob(B)=20%X20%=4%

Therefore the probability of drawing a red ball from both boxes is 4% i.e. 1 in 25 turns.

If we threw two dice at the same time, what would be the probability of two fours?

prob(4 *and 4*) = prob(4) X prob(4)= 16.7%X 16.7%=1/36 = 2.8%

What is the probability that the next two people you meet on the street were both born on a Friday?

prob(Friday and Friday) = prob (Friday) X prob (Friday) = 1/7 X 1/7= 2%

Proportion of different blood groups are O= 46%; A= 43%; B= 8%; AB= 3%

This means that the probability of finding a donor with blood group O is 46% and of A is 43% and so on. Now, what is the probability that the next <u>two</u> blood donors are both group O?

If in a labor room the probability of birth of a girl is 48% and the probability of birth of a boy is 52%, what is the probability that the first <u>three</u> babies born in this hospital are boys?

The probability that the next three babies born in the hospital are boys is:
prob(BBB) = 52% x 52% x 52% = 14%