

Significance test for a mean

How can we decide that value of a parameter (e.g. a mean) is truly different in two or more situations? Frequently in biostatistics we compare two groups or samples and we want to know whether these samples are truly different in relation to some parameters. For example suppose we measured the Hb level of boys and girls and found that mean Hb of boys was 13.5 and Hb of girls was 12.5. Do boys really have a higher Hb than girls? Is this difference real or it is due to chance? To answer this and similar questions we need to do a significance test (also called a hypothesis test).

To calculate a significance test for a parameter, we first have to assume what we think about the parameter by formulating two hypotheses i.e. the null hypothesis and the alternative hypothesis.

Hypothesis

Hypothesis is a statement of belief about the value of a population parameter which could be either true or false. For example we believe that the mean height of the male students in the technical college is 167cm. This is a hypothesis which could be either true or false. When we calculate significance test we have two hypothesis and want to determine which one is true that is why significance testing is also called hypothesis testing. These 2 hypotheses are the null hypothesis and the alternative hypothesis.

The null hypothesis (H_0)

The null hypothesis is the hypothesis that that the population parameter has a particular value. For example in comparison of the HB level of boys and girls the null hypothesis is the hypothesis that believes boys and girls have the same Hb level i.e. there is no difference between boys and girls. In the comparison of the effect of two drugs on lowering blood sugar, the null hypothesis says that the two drugs have the same effect, they are not different.

The alternative hypothesis(H_1)

The alternative hypothesis is the hypothesis that rejects the null hypothesis, that believes the null hypothesis is false. The alternative hypothesis believes that the parameter has another specified value or a specified range of values different from what the null hypothesis says. For example in the Hb example, the (H_1) believes that Hb of boys is not the same as that of girls i.e. the mean Hb of boys and girls is different. The alternative hypothesis is usually the hypothesis which the research believes is true and wants to prove. For example a company discovers a new drug for diabetes and believes that it lowers the blood sugar (H_1), the company may even believe that new drug is better than an old drug (H_1). The null hypothesis in these cases is that the new drug does not lower blood sugar or in the second situation is similar to the old drug.

Significance test for a mean: Example

Let's assume that the mean height of Kurdish men is 170 cm according to previous studies. We measured height of 144 students of the medical school and observed that their mean height was 168 cm with a standard deviation of 40 cm. We want to know whether the height of the medical students is similar to the population or not. How will we do that?

It is clear from the results that the **observed** height of medical students (168 cm) is not the same as that of the Kurdish men (170 cm). But we are not sure that this difference is true or not; this difference could be due to chance. Therefore we have to do a significance test to be sure.

The null hypothesis (H_0) is that the **observed** mean height of the students (X) is equal to the **hypothesised mean** (μ_0) which is the population mean in this example, i.e.

$$H_0: x = \mu_0$$

The alternative hypothesis (H_1) is that the mean height of the students (x) is not equal to the hypothesised mean (μ_0) which is the population mean in this example, i.e.

$$H_1: x \neq \mu_0$$

The significance test will assess the alternative hypothesis by calculating the probability of obtaining the observed value (the value which we have got from the study) if the null hypothesis were true. In case of a mean, the significance test will calculate the probability of obtaining a value for the observed mean if the null hypothesis were true. This is done by a test called z test if the sample is big or a student's t test if the sample is small. Student's t test could be used for large samples as well.

The formula for a z test is

$$z = \frac{\text{observed mean} - \text{hypothesised mean}}{\text{standard error (observed mean)}}$$

$$z = (\bar{x} - \mu_0) / SE(\bar{x})$$

The formula for t test is the same as z test

$$t \text{ test statistic} = \frac{\bar{x} - \mu_0}{SE(\bar{x})}$$

Now we go back to the earlier example.

Observed mean height of students= 168 cm; standard deviation=40 cm;
 sample=144
 Hypothesized mean height=170 cm

We want to know whether the height of the medical students is similar to the population or not. How will we do that?

Null hypothesis: observed height is equal to hypothesized height

$$H_0: x = \mu_0$$

Alternative hypothesis: observed height is not equal to hypothesized height

$$H_1: x \neq \mu_0$$

This sample(n) is 144 which is big so we can use z test

$$z = (\bar{x} - \mu_0) / SE(\bar{x})$$

We have everything in the equation except the standard error (SE) of the observed mean. We have to derive it from the SD and sample size (n).

SD=40 cm; n=144:

$$SE = SD / \sqrt{n}$$

$$SE = 40\text{cm} / \sqrt{144} = 40/12 = 3.3 \text{ cm}$$

No we can calculate z test from the equation

$$z = (\bar{x} - \mu_0) / SE(\bar{x})$$

. We know the observed mean height of medical school students is 168 cm and the hypothesized mean is 170 cm

$$z = \frac{\text{observed mean} - \text{hypothesised mean}}{\text{standard error (observed mean)}}$$

$$z = (168 - 170) / 3.3 = -2 / 3.3 = -0.60$$

What is this? This is the value of z statistics which is the standardized deviation from the hypothesized mean. We read the probability corresponding to this value of z from the table of normal distribution.

The value corresponding to z=0.6 in the two tailed values of the table of normal distribution is 0.549 or 0.55. What is this?

This is also called tow-tailed **p value**. This value tells as the probability of obtaining a value similar to the observed value or more extreme than it, if the null hypothesis were true. A **p value** of 0.55 means that there is a 55% probability that the students height is different from the hypothesized (population) height. Since this probability is very big we say the students' height is similar to the population height.

Table A1 Standard Normal distribution.

z	2-tailed P-value
0.0	1.000
0.1	0.920
0.2	0.841
0.3	0.764
0.4	0.689
0.5	0.617
0.6	0.549
0.7	0.484
0.8	0.424
0.9	0.368
1.0	0.317
1.1	0.271
1.2	0.230
1.3	0.194
1.4	0.162
1.5	0.134
1.6	0.110
1.7	0.089
1.8	0.072

In other words the height of the medical school students is not different from the height of the population.

A **p value** is the probability of obtaining a value equal to or more extreme than the value of the parameter we have estimated, if the null hypothesis were true. The larger the p the more similar the two means which we are comparing. The smaller the p value the more different the two means we are comparing. Because a smaller p value means that the probability of similarity is less.

Depending on the value of p, we either accept or reject the null hypothesis. If the p value is small we reject the null hypothesis, if the p is large we accept the null hypothesis. But how small should the p value be to reject the null hypothesis. The smaller the better but traditionally p value of 0.05 (5%)

Large p value	Small p value
Accept the null hypothesis (H_0) Reject the alternative hypothesis (H_1)	Reject null hypothesis (H_0) Accept alternative hypothesis (H_1)
The observed mean is not really (significantly) different from the hypothesized mean	The observed mean is really (significantly) different from the hypothesized mean
The apparent difference between the two means is due to random variation	The difference between the two means is real, it is not due to random variation

